MATHEMATICS (SYLLABUS D) 4024/22
Paper 2
October/November 2016
2 hours 30 minutes

Candidates answer on the Question Paper.

Additional Materials: Geometrical instruments
                          Electronic calculator

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO NOT WRITE IN ANY BARCODES.

Section A
Answer all questions.

Section B
Answer any four questions.

If working is needed for any question it must be shown in the space below that question.
Omission of essential working will result in loss of marks.
You are expected to use an electronic calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For \( \pi \), use either your calculator value or 3.142, unless the question requires the answer in terms of \( \pi \).

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.
The basic price of the 2016 model of a car is $21000. Sayeed and Rasheed each buy this model of car.

(a) (i) Sayeed pays a deposit of $756.

Calculate the deposit Sayeed pays as a percentage of the basic price.

\[ \text{Answer} \quad \text{......................................} \% \quad [1] \]

(ii) He then pays 24 monthly payments of $922.25.

Calculate the total amount that Sayeed pays as a percentage of the basic price.

\[ \text{Answer} \quad \text{......................................} \% \quad [2] \]
(b) Rasheed pays a deposit of $381 followed by 36 equal monthly payments. The total amount that he pays is 127% of the basic price of $21 000.

Calculate Rasheed’s monthly payment.

Answer $ ........................................ [3]

(c) $21 000 represented an increase of 5% on the basic price of the 2015 model.

Calculate the difference between the basic prices of the 2015 and 2016 models.

Answer $ ........................................ [3]
2 (a) Simplify $\frac{3a^2}{10bc} \div \frac{9a}{5b^2c}$.

Answer ........................................ [2]

(b) Simplify $\frac{h - k}{5h - 5k}$.

Answer ........................................ [2]

(c) Factorise $9m^2 - 4n^2$.

Answer ........................................ [1]
(d) Factorise \( q(p - 2) + 3(2 - p) \).

\[ \] Answer ........................................... [2]

(e) (i) Find the two solutions of \( 5x - 1 = \pm 9 \).

\[ \] Answer \( x = \) .............. or .............. [2]

(ii) The solutions of \( 5x - 1 = \pm 9 \) are also the solutions of \( 5x^2 + Bx + C = 0 \), where \( B \) and \( C \) are integers.

Find \( B \) and \( C \).

\[ \] Answer \( B = \) ............... , \( C = \) ............... [2]
3  (a) Complete the table of values for \( y = \frac{x}{20}(x^2 - 10) \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>0</td>
<td>-0.45</td>
<td>-0.6</td>
<td>-0.15</td>
<td>1.2</td>
<td></td>
</tr>
</tbody>
</table>

(b) Using a scale of 2 cm to 1 unit on both axes, draw the graph of \( y = \frac{x}{20}(x^2 - 10) \) for \( 0 \leq x \leq 5 \).

(c) By drawing a tangent, estimate the gradient of the curve at the point where \( x = 2.5 \).

Answer ..........................................

[1] [2]
(d) Use your graph to solve the equation \( \frac{x}{20}(x^2 - 10) = 0 \) for \( 0 \leq x \leq 5 \).

Answer \( x = \cdots \) or \( \cdots \) [2]

(e) The graph of \( y = \frac{x}{20}(x^2 - 10) \), together with the graph of a straight line \( L \), can be used to solve the equation \( x^3 + 10x - 80 = 0 \) for \( 0 \leq x \leq 5 \).

(i) Find the equation of line \( L \).

Answer \( \cdots \) [2]

(ii) Draw the graph of line \( L \) on the grid. [1]

(iii) Hence solve the equation \( x^3 + 10x - 80 = 0 \) for \( 0 \leq x \leq 5 \).

Answer \( x = \cdots \) [1]
In the framework $ABCD$, $BD = 3$ m. $BD \hat{A} = 27^\circ$, $BCD = 41^\circ$. $D \hat{B}C$ and $D \hat{A}B$ are right angles.

(i) Find $AD$.

Answer ................................. m [2]

(ii) Find $CD$.

Answer ................................. m [3]
(b) In triangle $PQR$, $PQ = 3\, \text{m}$ and $QR = 5\, \text{m}$. The area of triangle $PQR = 6\, \text{m}^2$.

Find the two possible values of $P\hat{Q}R$. 

Answer $P\hat{Q}R = \ldots\ldots\ldots\ldots$ or $\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ [3]
In the diagram, $A$ and $B$ are the centres of two circles that touch at $P$. The line $ACT$ touches the small circle at $T$ and intersects the large circle at $C$. $D$ is the point on $AB$ such that $CDA = 90^\circ$.

(a) Complete the following, to show that triangle $ACD$ is similar to triangle $ABT$.

In triangle $ACD$ and triangle $ABT$

- $\angle DAC = \angle ..................$ (same angle)
- $\angle CDA = \angle ..................$ (..............................................................)
- $\angle ACD = \angle ..................$ (two angles in a triangle are equal, so the third angles are equal)

Because the three pairs of angles are equal, the triangles are similar. [2]

(b) Given that the radii of the circles are 7 cm and 3 cm, calculate $CD$.

Answer ................................ cm [3]
6 \[ \begin{pmatrix} 2 & 0 \\ 3 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix} \]

(a) Find \( A + 2B \).

\[
\begin{pmatrix} \text{Answer} \\ \end{pmatrix}
\]

[2]

(b) Find \( AB \).

\[
\begin{pmatrix} \text{Answer} \\ \end{pmatrix}
\]

[2]

(c) \( A \begin{pmatrix} x \\ 2 \end{pmatrix} = \begin{pmatrix} 8 \\ 2y \end{pmatrix} \)

Find \( x \) and \( y \).

\[
\begin{pmatrix} \text{Answer} x = \ldots \ldots \ldots \ldots \quad y = \ldots \ldots \ldots \ldots \end{pmatrix}
\]

[2]

(d) Find \( B^{-1} \).

\[
\begin{pmatrix} \text{Answer} \\ \end{pmatrix}
\]

[2]
Section B [48 marks]

Answer four questions in this section.

Each question in this section carries 12 marks.

7 (a) \( x = \sqrt{a^2 + b^2} \)

(i) Calculate \( x \) when \( a = -0.73 \) and \( b = 1.84 \).

\[ \text{Answer } \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots [1] \]

(ii) Express \( b \) in terms of \( x \) and \( a \).

\[ \text{Answer } \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots [2] \]

(b)

\[ \begin{align*}
A & \quad B && C \\
& \quad x & \quad D \\
& \quad P & \quad Q \\
& \quad x+5 \quad R & \quad S
\end{align*} \]

\( ABCD \) and \( PQRS \) are rectangles.
\( AD = x \) cm and \( PS = (x + 5) \) cm.
Each rectangle has an area of 17 cm\(^2\).

(i) Write down an expression for \( PQ \) in terms of \( x \).

\[ \text{Answer } \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots [1] \]
(ii) \( AB \) is 3 cm longer than \( PQ \).

Form an equation in \( x \) and show that it simplifies to \( 3x^2 + 15x - 85 = 0 \).

(iii) Solve the equation \( 3x^2 + 15x - 85 = 0 \).

Give your solutions correct to 3 significant figures.

\[ \text{Answer } x = \ldots \text{ or } \ldots \] [3]

(iv) Find the perimeter of the rectangle \( PQRS \).

\[ \text{Answer } \ldots \text{ cm} \] [2]
8 (a) P and Q are points on the circumference of a circle, centre O, radius R cm. The minor arc \( PQ = 20 \) cm and \( \hat{POQ} = 48^\circ \).

(i) Show that \( R = 23.9 \), correct to one decimal place.

(ii) Calculate the area of the minor sector \( POQ \).

Answer ................................... \( \text{cm}^2 \) [2]

(iii) The minor sector \( POQ \) is removed from the circle and the remaining major sector is shaped to form an open cone of radius \( r \) cm.

Calculate \( r \).

Answer \( r = \) ................................... [2]
(b) [The curved surface area of a cone is \( \pi rl \), where \( l \) is the slant height]

Cone A has radius 7.5 cm and height 4 cm.

(i) Calculate the curved surface area of cone A.

\[ \text{Answer} \quad \pi \times 7.5 \times 4 \quad \text{cm}^2 \] [3]

(ii) Cone B is geometrically similar to cone A.

The ratio \( \text{curved surface area of cone A} : \text{curved surface area of cone B} \) is \( 64 : 25 \).

Find the height of cone B.

\[ \text{Answer} \quad \text{height of cone B} \quad \text{cm} \] [2]
ABCD is a level playing field.
\(AB = 65\, \text{m}, \ BC = 70\, \text{m}\) and \(CA = 110\, \text{m}\).
\(\angle DCA = 70^\circ, \angle DAC = 58^\circ\) and \(C\) is due South of \(B\).

(a) Calculate the bearing of \(A\) from \(C\).

Answer ........................................... [4]

(b) Calculate \(AD\).

Answer ........................................... m [3]
(e) There are two vertical trees, $AX$ and $CY$, each of height 17 m, one at each end of the path $AC$.

(i) Calculate the angle of elevation of $Y$ from $B$.

(ii) A bird flies in a straight line from $X$ to $Y$.
It takes 24 seconds.

Calculate the average speed of the bird.
Give your answer in kilometres per hour.

Answer ........................................  [2]

Answer ....................................... km/h [3]
10 (a)

$ACB$ and $OCD$ are straight lines.
$AC : CB = 1 : 2$.
$\overrightarrow{OA} = 3a$ and $\overrightarrow{OB} = 6b$.

(i) Express $\overrightarrow{AB}$ in terms of $a$ and $b$.

Answer ........................................  [1]

(ii) Express $\overrightarrow{AC}$ in terms of $a$ and $b$.

Answer ........................................  [1]

(iii) $\overrightarrow{BD} = 5a - b$.

Showing your working clearly, find $OC : CD$.

Answer .................... : ............... [4]
(b) 

(i) Describe fully the **single** transformation that maps triangle $A$ onto triangle $B$.

*Answer* .................................................................................................................................. [2]

(ii) Triangle $A$ is mapped onto triangle $C$ by the shear $H$ in which the $y$-axis is invariant, and $H(2, 1) = (2, 3)$.

(a) On the grid, draw and label triangle $C$. [2]

(b) State the shear factor of $H$.

*Answer* .......................................... [1]

(c) Find the matrix that represents $H$.

*Answer*  \[
\begin{pmatrix}
\end{pmatrix}
\] [1]
11 (a) Six hundred candidates took a mathematics examination which consisted of two papers. Each paper was marked out of 100.
The diagram shows, on the same grid, the cumulative frequency curves for Paper 1 and Paper 2.

(i) Use the cumulative frequency curve for Paper 1 to find an estimate of

(a) the median,

Answer ........................................... [1]

(b) the interquartile range,

Answer ........................................... [2]

(c) the number of candidates who scored more than 45.

Answer ........................................... [1]
(ii) A candidate scored 60 on Paper 1.

Using both graphs, estimate this candidate’s mark on Paper 2.

Answer ........................................... [1]

(iii) State, with a reason, which you think was the more difficult paper.

Answer Paper ............ because ............................................................................................

.............................................................................................................................................. [1]

Question 11(b) begins on the next page
(b) Amira has three $1 coins and two 20c coins in her purse. She picks out coins at random, one after the other. The coins are not replaced.

The tree diagram shows the possible outcomes and their probabilities when picking out two coins.

(i) Find $x$.

\[ x = \frac{3}{5} \times \frac{2}{5} + \frac{3}{5} \times \frac{3}{4} + \frac{2}{5} \times \frac{2}{4} = \frac{6}{25} + \frac{9}{20} + \frac{1}{5} = \frac{24}{100} + \frac{45}{100} + \frac{20}{100} = \frac{89}{100} \]

Answer \[ \frac{89}{100} \] [1]

(ii) Find the probability that the total value of the two coins picked out is 40 cents.

\[ \frac{6}{25} + \frac{9}{20} + \frac{1}{5} \]

Answer \[ \frac{24}{100} + \frac{45}{100} + \frac{20}{100} = \frac{89}{100} \] [1]

(iii) Find the probability that the total value of the two coins picked out is $1.20.

\[ \frac{3}{5} \times \frac{2}{5} + \frac{3}{5} \times \frac{3}{4} + \frac{2}{5} \times \frac{2}{4} = \frac{6}{25} + \frac{9}{20} + \frac{1}{5} = \frac{24}{100} + \frac{45}{100} + \frac{20}{100} = \frac{89}{100} \]

Answer \[ \frac{89}{100} \] [2]
At a car park, the charge is $1.40.
Amira picks out three coins, one after the other.

Find the probability that the total value of the three coins is $1.40.

Answer ........................................  [2]