Cambridge International Examinations
Cambridge International General Certificate of Secondary Education

CAMBRIDGE INTERNATIONAL MATHEMATICS
Paper 6 (Extended)
October/November 2015
1 hour 30 minutes

Candidates answer on the Question Paper.
Additional Materials: Graphics calculator

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, glue or correction fluid.
You may use an HB pencil for any diagrams or graphs.
DO NOT WRITE IN ANY BARCODES.

Answer both parts A and B.
You must show all relevant working to gain full marks for correct methods, including sketches.
In this paper you will also be assessed on your ability to provide full reasons and communicate your mathematics clearly and precisely.
At the end of the examination, fasten all your work securely together.
The total number of marks for this paper is 40.
A INVESTIGATION

STARS (20 marks)

You are advised to spend no more than 45 minutes on this part.

This investigation looks at how stars can be drawn by a robot ant.

Each diagram in this investigation is the path that the ant draws by repeating these two steps.

- move one unit forward
- turn anticlockwise through $A^\circ$ where $A < 180$

1 (a)

The ant repeats the two steps 7 times, going round the polygon until it reaches its starting position. The ant makes 1 complete revolution ($360^\circ$) to draw this regular 7-sided polygon.

Show that $A = 51.4$, correct to 1 decimal place.

(b) The ant draws a regular $n$-sided polygon.

Write down a formula for $A$ in terms of $n$. 

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2 (a) The diagrams show how the ant draws a 7-pointed star.

During the 4th turn the ant completes its first revolution.

Here is the completed 7-pointed star.

To draw a 7-pointed star, the ant must repeat the two steps 7 times, as shown in the diagrams. In doing so, the ant makes 2 complete revolutions (720°).

Calculate $A$. 

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The diagram shows another 7-pointed star. To draw it, the ant repeats the two steps 7 times.

(i) In drawing this star, how many complete revolutions does the ant make before it reaches its starting position?

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(ii) Explain why \( A = \frac{3 \times 360}{7} \).

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(iii) To draw a 7-pointed star, \( \frac{3 \times 360}{7} \) is the largest possible value of \( A \) which is less than 180°.

Show that 4 complete revolutions do not give a suitable value for \( A \).
3

The diagram shows the 5-pointed star drawn using the largest possible value of $A$ which is less than 180°. To draw it, the ant repeats the two steps 5 times and makes 2 complete revolutions.

Write down the calculation to find $A$.
Give your answer in a similar form to question 2(b)(ii).

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4 (a) Complete the table for stars that are drawn using the largest possible value of $A$ which is less than 180°. The first row gives a polygon and is shown here to help you see the pattern.

<table>
<thead>
<tr>
<th>Number of points on the star</th>
<th>Number of revolutions, $n$, that the ant makes</th>
<th>Calculation of $A$</th>
<th>$A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1</td>
<td>$\frac{1}{3} \times 360$</td>
<td>120</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>$\frac{3}{7} \times 360$</td>
<td>144</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td>154.3</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
<td>163.6</td>
</tr>
</tbody>
</table>

(b) Find a formula for $A$ in terms of $n$.

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(e) Find the number of points on the star when $A = 172.8$.

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5 **Question 2** shows the only two possible 7-pointed stars.

(a) Find the number of possible 11-pointed stars.

Write down the number of complete revolutions that the ant makes to draw each of these stars.

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(b) Explain why the calculation of $A$ for a 15-pointed star using 6 complete revolutions gives the calculation for the 5-pointed star.

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(c) Find all the values of \( A \) that give a 15-pointed star.
B MODELLING BODY MASS (20 marks)

You are advised to spend no more than 45 minutes on this part.

This task looks at three different models for the approximate body mass of an adult.

1 Here is a simple model to calculate the approximate body mass of an adult.

Measure the height in centimetres and subtract 100 to get the approximate mass in kilograms.

(a) Find the approximate body mass of an adult who is 1.8 m tall.

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(b) Find the height of an adult who has an approximate body mass of 50 kg.

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(c) For a height, \( h \) metres, the approximate body mass is \( M \) kilograms.
    Find the formula for \( M \) in terms of \( h \).

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(d) Sketch, showing appropriate scales, the graph of the approximate body mass, \( M \).
In the 19th century, the Belgian mathematician Quetelet suggested a different model.

In his model the approximate body mass of an adult, \( M \) kg, varies as the square of the height, \( h \) metres. For an adult with height 2 m the approximate body mass is 88 kg.

(a) Show that Quetelet’s model can be written as \( M = 22h^2 \).

(b) Show that an adult with height 1.5 m has an approximate body mass of 49.5 kg.

(c) An adult has an approximate body mass of 77 kg. Calculate the height of this adult.
3 (a) For some adults the simple model and Quetelet’s model give the same approximate body mass. Find the height of these adults.

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(b) For most adults, which model gives a larger approximate body mass? Explain your answer.
Quetelet’s model gives an approximate body mass which is too small for taller people.

A modern model uses $M = kh^n$ and the following information.

The approximate body mass of an adult is

- 78 kg for a height of 1.84 m
- 50 kg for a height of 1.54 m.

(a) Write down two equations in terms of $k$ and $n$.

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(b) Show that $1.56 = 1.195^n$, correct to 3 decimal places.

(c) Show that $n = 2.5$, correct to one decimal place.

(d) Using $n = 2.5$, calculate the value of $k$, correct to the nearest integer.

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Questions 4(e) and 5 are printed on the next page.
(e) Sketch the graph of the modern model.

5 Above which height does the modern model give a greater approximate body mass than the Quetelet model?

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