Cambridge International Examinations
Cambridge International General Certificate of Secondary Education

Cambridge INTERNATIONAL MATHEMATICS
Paper 5 (Core)
October/November 2014
1 hour

Candidates answer on the Question Paper.
Additional Materials: Graphics Calculator

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, glue or correction fluid.
You may use an HB pencil for any diagrams or graphs.
DO NOT WRITE IN ANY BARCODES.

Answer all the questions.
You must show all relevant working to gain full marks for correct methods, including sketches.
In this paper you will also be assessed on your ability to provide full reasons and communicate your mathematics clearly and precisely.
At the end of the examination, fasten all your work securely together.
The total number of marks for this paper is 24.

This document consists of 6 printed pages and 2 blank pages.
Answer all the questions.

INVESTIGATION CUBES

Identical small cubes fit together to make larger cubes. There are no gaps between these small cubes. For each cube that is made, a cross is marked on each outside face of each small cube.

The diagram shows the first three cubes that can be made.

Diagram 1
1 by 1 by 1

Diagram 2
2 by 2 by 2

Diagram 3
3 by 3 by 3

This investigation is about the number of crosses that can be marked on cubes.

Look at the 1 by 1 by 1 cube. It is made from 1 small cube. It has 6 crosses on it (3 crosses are on the faces not seen on the diagram).

1 Look at the 2 by 2 by 2 cube.

(a) How many small cubes is this cube made from?
...............................................................................................................

(b) Explain why there are only 3 crosses on each small cube.
................................................................................................................................................

(c) Find the total number of crosses on the 2 by 2 by 2 cube.
.............................................................................................................
2 Look at the 3 by 3 by 3 cube.

(a) How many small cubes is this cube made from?

............................................

(b) How many of these small cubes have 3 crosses on them?

............................................

(c) There are 12 small cubes with 2 crosses on them.
    There is 1 small cube with no crosses on it.

How many small cubes have only 1 cross on them?

.............................................

3 (a) On the dotty grid below, draw a 4 by 4 by 4 cube.

Mark a cross on the outside face of each small cube.

.............................................

(b) The 4 by 4 by 4 cube is made from 64 small cubes.

    (i) How many of these small cubes have 3 crosses on them?

.............................................

    (ii) How many of these small cubes have 2 crosses on them?

.............................................
4 Complete this table. You may use the dotty grid on page 6 to help you.

<table>
<thead>
<tr>
<th>Size of cube</th>
<th>Total number of small cubes</th>
<th>Number of small cubes with</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0 crosses</td>
</tr>
<tr>
<td>2 by 2 by 2</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>3 by 3 by 3</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>4 by 4 by 4</td>
<td>64</td>
<td>8</td>
</tr>
<tr>
<td>5 by 5 by 5</td>
<td></td>
<td>27</td>
</tr>
</tbody>
</table>

5 (a) To work out the number of crosses on the 3 by 3 by 3 cube, complete the following.

1 small cube with 0 crosses gives 0 crosses

................ small cubes with 1 cross gives 6 crosses

12 small cubes with 2 crosses gives ............... crosses

................ small cubes with 3 crosses gives ............... crosses

Total = ............... crosses

(b) The total number of crosses can also be worked out by the following method. Complete the following.

The number of crosses on one face of the 3 by 3 by 3 cube is .................

So the total number of crosses on all the 6 faces is .................

(c) Find the total number of crosses on a 4 by 4 by 4 cube.

.............................................
The number of small cubes with 0 crosses forms a sequence of cube numbers.

<table>
<thead>
<tr>
<th>Size of cube</th>
<th>2 by 2 by 2</th>
<th>3 by 3 by 3</th>
<th>4 by 4 by 4</th>
<th>5 by 5 by 5</th>
<th>( n ) by ( n ) by ( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of small cubes with 0 crosses</td>
<td>0</td>
<td>1</td>
<td>8</td>
<td>27</td>
<td>( n ) by ( n ) by ( n )</td>
</tr>
</tbody>
</table>

For an \( n \) by \( n \) by \( n \) cube, find an expression, in terms of \( n \), for the number of small cubes with 0 crosses. Write your answer in the table above.

(b) The number of small cubes with 1 cross forms a sequence. Find the \( n \)th term of this sequence.

(c) The number of small cubes with 2 crosses forms a sequence. Find the \( n \)th term of this sequence.