READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, glue or correction fluid.
You may use an HB pencil for any diagrams or graphs.
DO NOT WRITE IN ANY BARCODES.

Answer all the questions.
You must show all relevant working to gain full marks for correct methods, including sketches.
In this paper you will also be assessed on your ability to provide full reasons and communicate your mathematics clearly and precisely.
At the end of the examination, fasten all your work securely together.
The total number of marks for this paper is 24.
INVESTIGATION

ESTIMATING $\pi$

This investigation is about using relative frequency to estimate the value of $\pi$.

Area of rectangle = length $\times$ width.
Area, $A$, of circle radius $r$ is $A = \pi r^2$.

Lee draws circles on rectangular pieces of paper.
He drops grains of rice at random onto the pieces of paper.
He counts the number of grains of rice inside each circle.

1. Lee draws a circle of radius 5 cm on a rectangular piece of paper measuring 40 cm by 20 cm.

(a) (i) Find the area of the rectangle.

...................................................

(ii) The probability, $p$, that a grain of rice lands inside the circle is

\[ p = \frac{\text{area of circle}}{\text{area of rectangle}}. \]

The area of the circle is $25 \times \pi$.

Show that $p$ is approximately 0.098 for this piece of paper.

(b) Lee drops 10 grains of rice at random onto the piece of paper.
The diagram shows the result.

Key:
$\Rightarrow$ represents one grain of rice.
(i) How many of the grains of rice are inside the circle?

............................................................................

(ii) The relative frequency that a grain of rice is inside the circle

\[ \frac{\text{number of grains of rice inside the circle}}{\text{total number of grains of rice dropped}} \]

Find the relative frequency that a grain of rice is inside the circle.

............................................................................

(c) Lee drops 10 more grains of rice at random onto the piece of paper.

![Diagram of grains of rice and a circle](NOT TO SCALE)

Show that the relative frequency that a grain of rice is inside the circle is 0.15.

(d) The relative frequency that a grain of rice is inside the circle gives an estimate for the probability, \( p \).

The area of the circle is \( 25 \times \pi \).

Use \( \frac{\text{area of circle}}{\text{area of rectangle}} = 0.15 \) to show that an estimate for \( \pi \) is 4.8.
2 Lee draws a circle of radius 10 cm on a rectangular piece of paper measuring 30 cm by 20 cm.

(a) Complete this statement with a number.

Area of circle = .................. $\times \pi$

(b) Lee drops 10 grains of rice at random onto the piece of paper. Diagram A shows the result.

Lee removes the 10 grains of rice and drops another 10 grains of rice at random onto the piece of paper. Diagram B shows the result.
(i) Complete the table.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>Combined results for all 20 grains of rice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of grains of rice inside circle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative frequency</td>
<td></td>
<td></td>
<td>(\frac{10}{20})</td>
</tr>
</tbody>
</table>

(ii) Use the formula

\[
\frac{\text{area of circle}}{\text{area of rectangle}} = \text{relative frequency}
\]

...
3 Lee draws two circles on a different rectangular piece of paper. The circles touch the edges of the piece of paper and touch each other.

![Diagram of two circles touching each other and touching the edges of a rectangle]

(a) (i) Find the value of $x$.

...................................................

(ii) Complete this statement with a number.

**Total** area of the two circles = .................. $\times \pi$

(b) Lee drops 50 grains of rice at random onto the piece of paper. He removes the 50 grains of rice and drops another 50 grains of rice at random onto the piece of paper. The table shows his results.

<table>
<thead>
<tr>
<th>First 50</th>
<th>Second 50</th>
<th>Combined results for all 100 grains of rice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of grains of rice inside circle</td>
<td>36</td>
<td>43</td>
</tr>
<tr>
<td>Relative frequency</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Complete the table.

(c) Use the formula \[
\frac{\text{total area of circles}}{\text{area of rectangle}} = \text{relative frequency} \]
to estimate $\pi$.

\[
\pi = ...........................................
\]

(d) Give **one** reason why the estimate for $\pi$ in question 3(c) is more accurate than the estimate for $\pi$ in question 2(b)(ii).

...................................................................................................................................................................
4 Lee draws one circle of radius \( r \) cm on a different piece of paper. The circle touches all four edges of the paper.

(a) Lee drops 500 grains of rice at random onto the piece of paper. He removes the 500 grains of rice and drops another 500 grains of rice at random onto the piece of paper. The combined number of grains of rice inside the circle is 785.

Use the formula
\[
\frac{\text{area of circle}}{\text{area of rectangle}} = \text{relative frequency}
\]
to estimate \( r \).

\[
\pi = \text{relative frequency}
\]

(b) Complete this sentence with a single number.

To estimate the value of \( \pi \) when the circle touches all four edges of the paper, multiply the relative frequency by .................

(c) Lee drops \( n \) grains of rice at random onto the piece of paper. He removes the \( n \) grains of rice and drops another \( n \) grains of rice at random onto the piece of paper. The combined number of grains of rice inside the circle is \( k \).

Show that \( \frac{2k}{n} \) is an estimate for \( \pi \).