Cambridge IGCSE™

CAMBRIDGE INTERNATIONAL MATHEMATICS 0607/63
Paper 6 Investigation and Modelling (Extended)
October/November 2020
1 hour 40 minutes

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS
- Answer both part A (Questions 1 to 4) and part B (Questions 5 to 8).
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

INFORMATION
- The total mark for this paper is 60.
- The number of marks for each question or part question is shown in brackets [ ].

This document has 20 pages. Blank pages are indicated.
Answer both parts A and B.

A INVESTIGATION (QUESTIONS 1 TO 4)

AREAS OF POLYGONS INSIDE AND OUTSIDE A CIRCLE  (30 marks)

You are advised to spend no more than 50 minutes on this part.

This investigation looks at the areas of polygons drawn inside and outside a circle of radius 10 cm.

An inscribed polygon is a polygon in which all the vertices lie on a circle.
This is an inscribed square.

A circumscribed polygon is a polygon in which each side is a tangent to a circle.
This is a circumscribed square.

You may find some of these formulas useful.

Area, \( A \), of circle, radius \( r \) \[ A = \pi r^2 \]

Area, \( A \), of triangle, base \( b \), height \( h \) \[ A = \frac{1}{2} bh \]

In a right-angled triangle,

\[ \sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} \]

\[ \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} \]

\[ \tan \theta = \frac{\text{opposite}}{\text{adjacent}} \]
A square circumscribes a circle, centre $O$, radius 10 cm.

Work out the area of the square.
A square is inscribed in a circle, centre $O$, radius 10 cm.

Work out the area of the square.

\[ \text{Area of square} \]

\[ \text{Area of circle} \]

\[ \text{Area of circumscribed square} \]

Use this statement to complete the inequality below.

\[ \text{Area of inscribed square} < \text{Area of circle} < \text{Area of circumscribed square} \]

\[ \text{.................} \}

\[ \pi \}

\[ \text{.................} \]
A regular hexagon is inscribed in a circle, centre $O$, radius 10 cm.

Find the area of the hexagon.

.................................................  [3]
(b) (i) An equilateral triangle has height 10 cm.
Find the area of the triangle.

.................................................  [3]

(ii) A regular hexagon circumscribes a circle, centre $O$, radius 10 cm.
Using your answer to part (i), find the area of the hexagon.

.................................................  [2]
(c)  

(i)  Use Question 1(c), Question 2(a) and Question 2(b)(ii) to complete the inequality.

\[ \ldots \ldots < \pi < \ldots \ldots \]  \[1\]

(ii)  Give a geometric reason why the range in the inequality in Question 2(c)(i) is smaller than the range in the inequality in Question 1(d).

...........................................................................................................................................................................  \[1\]
A regular 12-sided polygon is inscribed in a circle, centre $O$, radius 10 cm.

Find the area of this polygon.

\[
\text{Area of the polygon} \quad \boxed{} \quad [2]
\]
A regular 12-sided polygon circumscribes a circle, centre $O$, radius 10 cm.

Find the area of this polygon.

(c) Use the answers to part (a) and part (b) to complete the inequality.

$\ldots < \pi < \ldots$ [1]
4  (a)  (i)  Show that a formula for the area, $A \text{cm}^2$, of a regular polygon with $n$ sides **inscribed** in a circle, radius 10 cm, is

$$A = 50n \sin\left(\frac{360}{n}\right)^\circ.$$ 

(ii)  Show that a formula for the area, $B \text{cm}^2$, of a regular polygon with $n$ sides that **circumscribes** a circle, radius 10 cm, is

$$B = 100n \tan\left(\frac{180}{n}\right)^\circ.$$ 

[2]
**(b) (i)** Work out the area of a regular polygon with 100 sides that is **inscribed** in a circle, radius 10 cm. Give your answer correct to 4 significant figures.

................................................. [2]

**(ii)** Work out the area of a regular polygon with 100 sides that **circumscribes** a circle, radius 10 cm. Give your answer correct to 4 significant figures.

................................................. [2]

**(c)** Use your answers to **part (b)** to explain how you can find the value of \( \pi \) correct to 3 significant figures.

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..................................................................................................................................................... [1]
B MODELLING (QUESTIONS 5 TO 8)

MODELLING CONTAINERS (30 marks)

You are advised to spend no more than 50 minutes on this part.

Olivia wants to design a closed container with a volume of 1000 cm$^3$ and minimum surface area.

5 Olivia uses a square-based cuboid to model the container.

(a) (i) Write down a formula for the volume of the cuboid, $V$ cm$^3$, in terms of $x$ and $h$.

................................................. [1]

(ii) Find a formula for the surface area, $S$ cm$^2$, of the cuboid, in terms of $x$ and $h$. Give your answer in its simplest form.

................................................. [2]

(b) (i) $V = 1000$.

Write $h$ in terms of $x$.

................................................. [1]

(ii) Show that $S = 2x^2 + \frac{4000}{x}$.

[1]
(iii) Work out the value of \( S \) when \( x = 25 \).

................................................. [1]

(c) Sketch the graph of \( S = 2x^2 + \frac{4000}{x} \) for \( 0 < x \leq 25 \).

(d) (i) Find the minimum surface area of the cuboid.

................................................. [1]

(ii) Describe the container that gives the minimum surface area for Olivia’s model.
Volume, $V$, of a cylinder of radius $r$, height $h$
\[ V = \pi r^2 h \]
Curved surface area, $A$, of a cylinder of radius $r$, height $h$
\[ A = 2\pi rh \]

Olivia now uses a cylinder to model the container.

The total surface area of this model is $T\text{ cm}^2$.

(a) $V = 1000$.

Show that
\[ T = 2\pi r^2 + \frac{2000}{r} \]

(b) (i) Find the minimum surface area of the cylinder.
(ii) Find the dimensions of the cylinder with the minimum surface area.

\[ r = \ldots \] \hspace{1cm} \[ h = \ldots \] \hspace{1cm} [2]
Volume, \( V \), of a pyramid, base area \( A \), height \( h \) \[ V = \frac{1}{3}Ah \]

Olivia now uses a square-based pyramid to model the container.

The pyramid, \( OABCD \), has a square base of side \( x \) cm and height \( h \) cm. The vertex of the pyramid, \( O \), is directly above the centre of the square base. \( E \) is the mid-point of \( BC \).

(a) Find an expression for \( OE \) in terms of \( h \) and \( x \).
(b) The total surface area of this model is $P \text{ cm}^2$.

$V = 1000$.

Show that $P = x^2 + \sqrt{x^6 + 36000000}$. 

(c) (i) Find the minimum surface area of the pyramid.

.................................................  [2]

(ii) Find the dimensions of the pyramid with the minimum surface area.

\[ x = \text{.................................} \]

\[ h = \text{.................................}  \]  [2]
8 Olivia recommends the container with the smallest surface area to a company.

Give a geometric reason why the company might not accept Olivia’s recommendation.

Olivia recommends the ....................................................

Geometric reason ........................................................................................................................................
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