MATHEMATICS
Paper 4 (Extended)
May/June 2021
2 hours 30 minutes

You must answer on the question paper.

You will need: Geometrical instruments

INSTRUCTIONS
● Answer all questions.
● Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
● Write your name, centre number and candidate number in the boxes at the top of the page.
● Write your answer to each question in the space provided.
● Do not use an erasable pen or correction fluid.
● Do not write on any bar codes.
● You should use a calculator where appropriate.
● You may use tracing paper.
● You must show all necessary working clearly.
● Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
● For π, use either your calculator value or 3.142.

INFORMATION
● The total mark for this paper is 130.
● The number of marks for each question or part question is shown in brackets [ ].

This document has 20 pages. Any blank pages are indicated.
1 (a) (i) Yasmin and Zak share an amount of money in the ratio 21 : 19.
Yasmin receives $6 more than Zak.

Calculate the total amount of money shared by Yasmin and Zak.

$ ...............................................  [2]

(ii) In a sale, all prices are reduced by 15%.

(a) Yasmin buys a blouse with an original price of $40.

Calculate the sale price of the blouse.

$ ...............................................  [2]

(b) Zak buys a shirt with a sale price of $29.75.

Calculate the original price of the shirt.

$ ...............................................  [2]
(b) Xavier’s salary increases by 2% each year. 
In 2010, his salary was $40 100.

(i) Calculate his salary in 2015.
   Give your answer correct to the nearest dollar.

   $ .................................................  [3]

(ii) In which year is Xavier’s salary first greater than $47 500?

   .................................................  [3]

(c) In January 2020, the population of a town was 5% more than its population in January 2018. 
In January 2021, the population of this town was 2% less than its population in January 2020.

   Calculate the overall percentage increase in the population from January 2018 to January 2021.

   ............................................. %  [2]
2 (a) \[ y = px^2 + t \]

(i) Find the value of \( y \) when \( p = 3 \), \( x = 2 \) and \( t = -13 \).

\[ y = \ldots................................................. \ [2] \]

(ii) Rearrange the formula to write \( x \) in terms of \( p \), \( t \) and \( y \).

\[ x = \ldots................................................. \ [3] \]

(b) (i) Factorise.

\[ 15x^2 - 2x - 8 \]

\[ \ldots................................................. \ [2] \]

(ii) Solve the equation.

\[ 15x^2 - 2x - 8 = 0 \]

\[ x = \ldots........... \text{ or } x = \ldots........... \ [1] \]

(e) Factorise completely.

\[ x^3 - 16xy^2 \]

\[ \ldots................................................. \ [3] \]
(d) Simplify: \[ \frac{2x-1 - 4ax + 2a}{2x^2 - x} \]
3 (a) Zoe’s test scores last term were 6, 7, 7, 7, 8, 9, 9, 10, 10.

Find

(i) the range,

............................................... [1]

(ii) the mode,

............................................... [1]

(iii) the median.

............................................... [1]

(b) The cumulative frequency diagram shows information about the time taken by each of 200 students to solve a problem.

Use the diagram to find an estimate of

(i) the median,

............................................... min [1]

(ii) the interquartile range.

............................................... min [2]
(c) The test scores of 200 students are shown in the table.

<table>
<thead>
<tr>
<th>Score</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>3</td>
<td>10</td>
<td>43</td>
<td>75</td>
<td>48</td>
<td>21</td>
</tr>
</tbody>
</table>

Calculate the mean.

.................................................  [3]

(d) The height, in cm, of each of 200 plants is measured. The histogram shows the results.

Calculate an estimate of the mean height. You must show all your working.

............................................ cm  [6]
4  (a)  $A$ is the point $(1, 5)$ and $B$ is the point $(3, 9).$

$M$ is the midpoint of $AB.$

(i)  Find the coordinates of $M.$

\[
(................... , ...................) \quad [2]
\]

(ii)  Find the equation of the line that is perpendicular to $AB$ and passes through $M.$

Give your answer in the form  $y = mx + c.$  

\[
y = .................. ................. \quad [4]
\]

(b)  The position vector of $P$ is $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ and the position vector of $Q$ is $\begin{pmatrix} -2 \\ 5 \end{pmatrix}.$

(i)  Find the vector $\overrightarrow{PQ}.$

\[
\left( \begin{array}{c}
\ \\
\end{array} \right) \quad [2]
\]

(ii)  $R$ is the point such that $\overrightarrow{PR} = 3 \overrightarrow{PQ}.$

Find the position vector of $R.$

\[
\left( \begin{array}{c}
\ \\
\end{array} \right) \quad [2]
\]
(i) Find $\overrightarrow{OY}$ in terms of $t$ and $u$.
Give your answer in its simplest form.

$$\overrightarrow{OY} = ................................................. [2]$$

(ii) $Z$ is on $OT$ and $YZ$ is parallel to $UO$.

Find $\overrightarrow{OZ}$ in terms of $t$ and/or $u$.
Give your answer in its simplest form.

$$\overrightarrow{OZ} = ................................................. [1]$$
5 Solve the simultaneous equations.

(a) \[ \begin{align*}
    x + 2y &= 13 \\
    x + 5y &= 22
\end{align*} \]

(b) \[ \begin{align*}
    y &= 2 - x \\
    y &= x^2 + 2x + 2
\end{align*} \]
6 In a class of 24 students, 18 students like homework ($H$), 15 students like tests ($T$) and 1 student does not like homework and does not like tests.

(a) Complete the Venn diagram to show this information.

(b) Write down the number of students who like both homework and tests.

(c) Find $n(H \cap T)$.

(d) A student is picked at random from the class.

Write down the probability that this student likes tests but does not like homework.

(e) Two students are picked at random from the class.

Find the probability that both students do not like homework and do not like tests.

(f) Two of the students who like homework are picked at random.

Find the probability that both students also like tests.
7  (a) 

![Number line with a solid dot at -1 and an open dot at 1.]

Write down the inequality in \( x \) shown by the number line.

................................................. [2]

(b) (i) Write \( x^2 + 4x + 1 \) in the form \( (x+p)^2 + q \).

................................................. [2]

(ii) Use your answer to part (b)(i) to solve the equation \( x^2 + 4x + 1 = 0 \).

\[ x = \ldots \ldots \ldots \text{ or } x = \ldots \ldots \ldots \] [2]
(iii) Use your answer to part (b)(i) to write down the coordinates of the minimum point on the graph of \( y = x^2 + 4x + 1 \).

\[ (..................,..................) \quad [2] \]

(iv) On the diagram, sketch the graph of \( y = x^2 + 4x + 1 \).
8 (a) A solid cuboid measures 20 cm by 12 cm by 5 cm.

(i) Calculate the volume of the cuboid.

.......................................... cm³ [1]

(ii) (a) Calculate the total surface area of the cuboid.

.......................................... cm² [3]

(b) The surface of the cuboid is painted.
The cost of the paint used is $1.52.

Find the cost to paint 1 cm² of the cuboid.
Give your answer in cents.

........................................ cents [1]

(b) A solid metal cylinder with radius $x$ and height $\frac{9x}{2}$ is melted.
All the metal is used to make a sphere with radius $r$.

Find $r$ in terms of $x$.

[The volume, $V$, of a sphere with radius $r$ is $V = \frac{4}{3}\pi r^3$.]

$r = ...........................................$ [3]
The diagram shows a cylinder of length 150 cm on horizontal ground. The cylinder has radius 20 cm. The cylinder contains water to a depth of 5 cm, as shown in the diagram.

Calculate the volume of water in the cylinder. Give your answer in litres.

........................................ litres [7]
Calculate the perimeter of the quadrilateral $ABCD$. 

\[ \text{............................................ cm [7]} \]
The diagram shows a cube. The length of the diagonal $AB$ is 8.5 cm.

(i) Calculate the length of an edge of the cube.

............................................ cm [3]

(ii) Calculate the angle between $AB$ and the base of the cube.

............................................. [3]
10 \hspace{1cm} \begin{align*} f(x) &= 3x - 2 \\ g(x) &= 5x - 7 \\ h(x) &= x^2 + x \\ j(x) &= 3^x \end{align*}

(a) Find

(i) \( f(2), \)

................................................................................. [1]

(ii) \( g(2), \)

................................................................................. [1]

(iii) \( gf(2). \)

................................................................................. [1]

(b) Find \( f^{-1}(x). \)

\[ f^{-1}(x) = \] ................................................................................. [2]

(c) Find \( hf(x), \) giving your answer in the form \( ax^2 + bx + c. \)

................................................................................. [3]

(d) Find the derivative of \( h(x). \)

................................................................................. [1]

(e) (i) Find \( x \) when \( j^{-1}(x) = 4. \)

\[ x = \] ................................................................................. [1]

(ii) Simplify \( j^{-1}j(x). \)

................................................................................. [1]
11 (a) These are the first four terms of a sequence.

11  7  3  −1

(i) Write down the next term.

................................................. [1]

(ii) Write down the term to term rule for this sequence.

................................................................. [1]

(iii) Find the $n$th term of this sequence.

......................................................... [2]

(b) The $n$th term of a different sequence is $\frac{2n}{n+1}$.

(i) Find the difference between the 5th term and the 6th term of this sequence.
Give your answer as a fraction.

......................................................... [2]

(ii) Is $\frac{3}{4}$ a term in this sequence?
Show how you decide.

[3]